PHYSICS 110A

Homework 1
Due in class, Tuesday, January 13.

1. Find the gradients of the following functions
   
   (a) \( f(x, y, z) = x^2 + y^3 + z^4 \).
   (b) \( f(x, y, z) = e^x \sin(y) \ln(z) \).

2. Show that
   
   (a) \( \nabla(r^2) = 2r \).
   (b) \( \nabla(1/r) = -\hat{r}/r^2 \).
   (c) What is the general formula for \( \nabla(r^n) \)?

3. Calculate the divergence of the following vector functions
   
   (a) \( x^2 \hat{x} + 3xz^2 \hat{y} - 2xz \hat{z} \).
   (b) \( xy \hat{x} + 2yz \hat{y} + 3zx \hat{z} \).

4. Calculate the curls of the vector functions in Qu. 3.

5. Calculate the Laplacian of the following functions:
   
   (a) \( T_a = x^2 + 2xy + 3z + 4 \).
   (b) \( T_b = \sin x \sin y \sin z \).
   (c) \( \mathbf{v} = x^2 \hat{x} + 3xz^2 \hat{y} - 2xz \hat{z} \).

6. (a) Prove that the divergence of a curl is always zero.
   (b) Prove that the curl of a gradient is always zero.

7. Griffiths problem 1.32. (Verification of divergence theorem.)

8. Griffiths problem 1.33. (Verification of Stokes’ theorem.)

9. Evaluate the following integrals
   
   (a) \( \int_0^6 (3x^2 - 2x - 1) \delta(x - 3) \, dx \).
   (b) \( \int_0^3 x^3 \delta(x + 1) \, dx \).
   (c) \( \int_{-3}^\infty \ln(x + 3) \delta(x + 2) \, dx \).

10. Show that
    
    \[ x \frac{d}{dx} \left( \delta(x) \right) = -\delta(x). \]
    
    Hint: Use integration by parts.